

# Bayesian Approach to Atmospheric Correction of Satellite Ocean-Color Imagery: Confidence Domains of the Retrieved Water Reflectance

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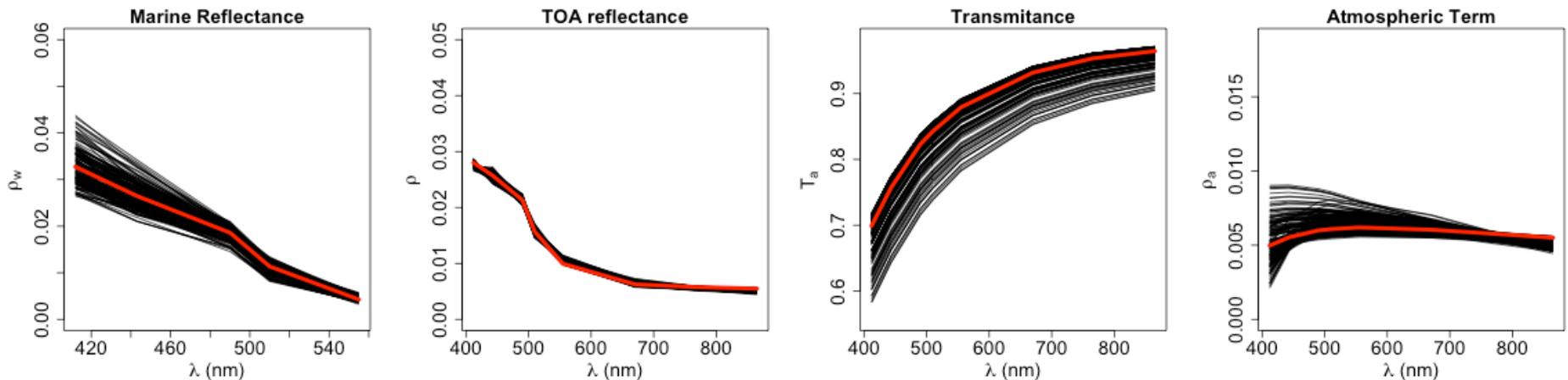
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## Ill-posed Nature of the inverse Problem

-The ocean color inverse problem (or atmospheric correction) is the retrieval of water reflectance from TOA reflectance.

-Multiple combinations of atmospheric and oceanic parameters (or pre-images) yield the same TOA reflectance. This places the inverse problem in a probabilistic context.

$$\rho \approx \rho_a + \rho_w T_a$$



*Example of pre-images. Actual values of  $\rho_w$ ,  $\rho$ ,  $T_a$ , and  $\rho_a$  are displayed in red, and the pre-images at a distance no more than  $\delta = 0.001$  are displayed in black. The search spaces for the pre-images include NOMAD and AERONET-OC data sets and maritime, continental, and urban aerosols in various proportions and amount.*

## Bayesian Methodology

-The forward model is written as:  $\rho = \phi(\rho_w, x_a) + \varepsilon$ , where  $\rho$  is the TOA reflectance,  $\rho_w$  is the water reflectance,  $x_a$  denote the atmospheric parameters, and  $\varepsilon$  is a random noise.

-In the Bayesian approach to inverse problems,  $\rho_w$  and  $x_a$  are treated as random variables. This defines a probabilistic model, where any vector of measurements  $\rho^{\text{obs}}$  is considered a realization of the random vector  $\rho$ .

-The probabilistic model is specified by the forward model together with the distributions of  $\varepsilon$  and of  $(\rho_w, x_a)$ . The distribution of  $(\rho_w, x_a)$ , called the prior distribution, describes in a probabilistic manner the prior knowledge one may have about  $\rho_w$  and  $x_a$  before the acquisition of the data.

## Bayesian Methodology (cont.)

-The Bayesian solution of the inverse problem of retrieving  $(\rho_w, x_a)$  from  $\rho$  is defined as the conditional distribution  $P[(\rho_w, x_a)/\rho]$ . It is called the posterior distribution. Hence, given the observation  $\rho^{\text{obs}}$ , the solution is expressed as the probability measure  $P[(\rho_w, x_a)/\rho = \rho^{\text{obs}}]$ .

-One is generally interested in certain relevant characteristics of the posterior distribution: its mean, which gives an estimate of the parameters to retrieve ( $\rho_w$  and  $x_a$ ), and its covariance, which provides an accompanying measure of uncertainty.

-One may also compute a p-value, i.e., the probability that  $\rho$  takes a value at least as extreme as  $\rho^{\text{obs}}$ . Since the whole procedure consists of inverting a forward model (a component of which is a RT model), the p-value allows one to detect situations for which the forward model is unlikely to explain the data.

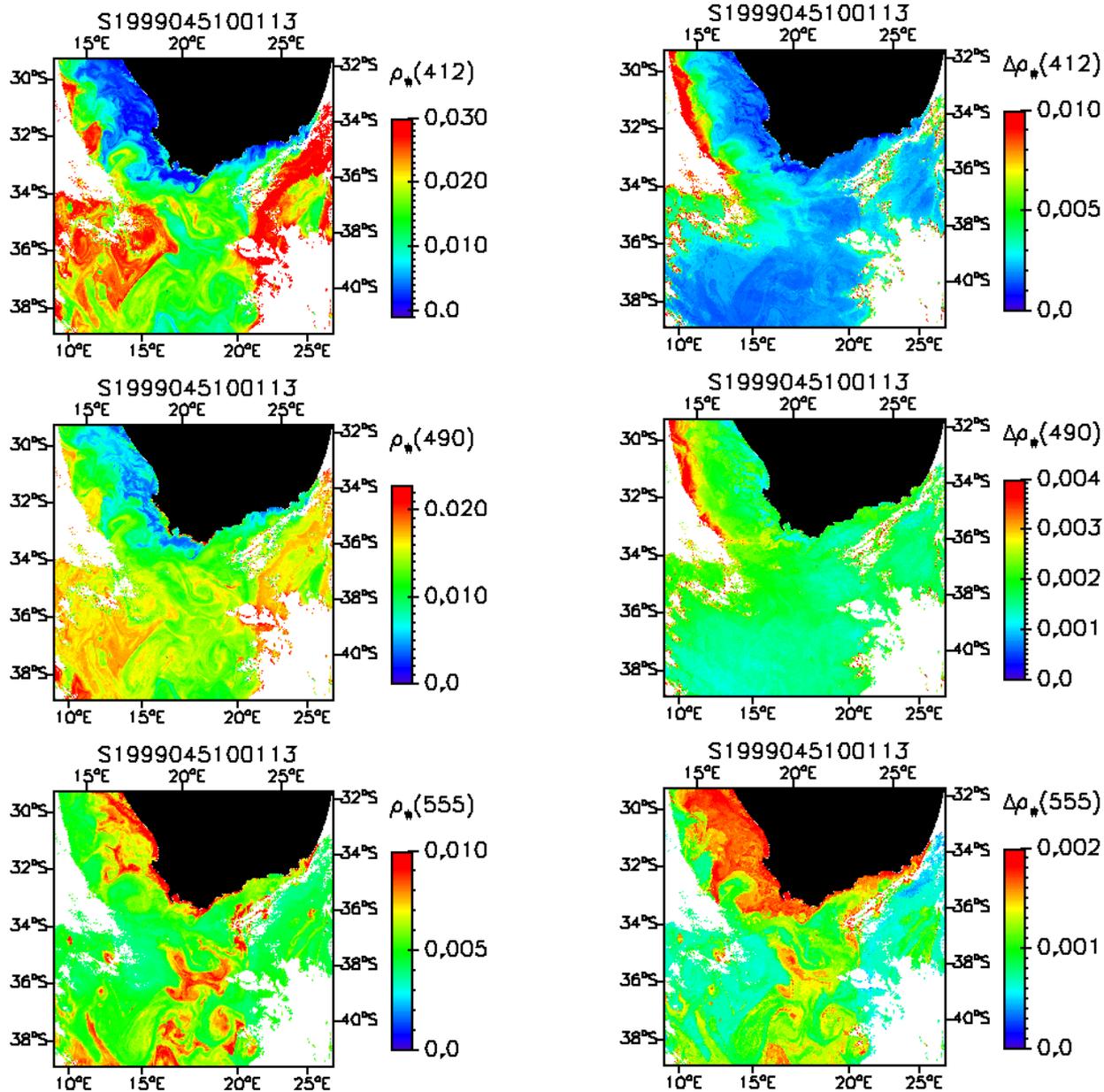
## Connection with the Classical Scheme

-Consider the conditional expectation  $E[\rho_w/\rho]$ . Since  $E[\rho_w/\rho] = E[E[\rho_w/\rho, x_a]/\rho]$ , we see that  $E[\rho_w/\rho, x_a]$  can be modeled first, and then averaged conditionally on  $\rho$  in a second time.

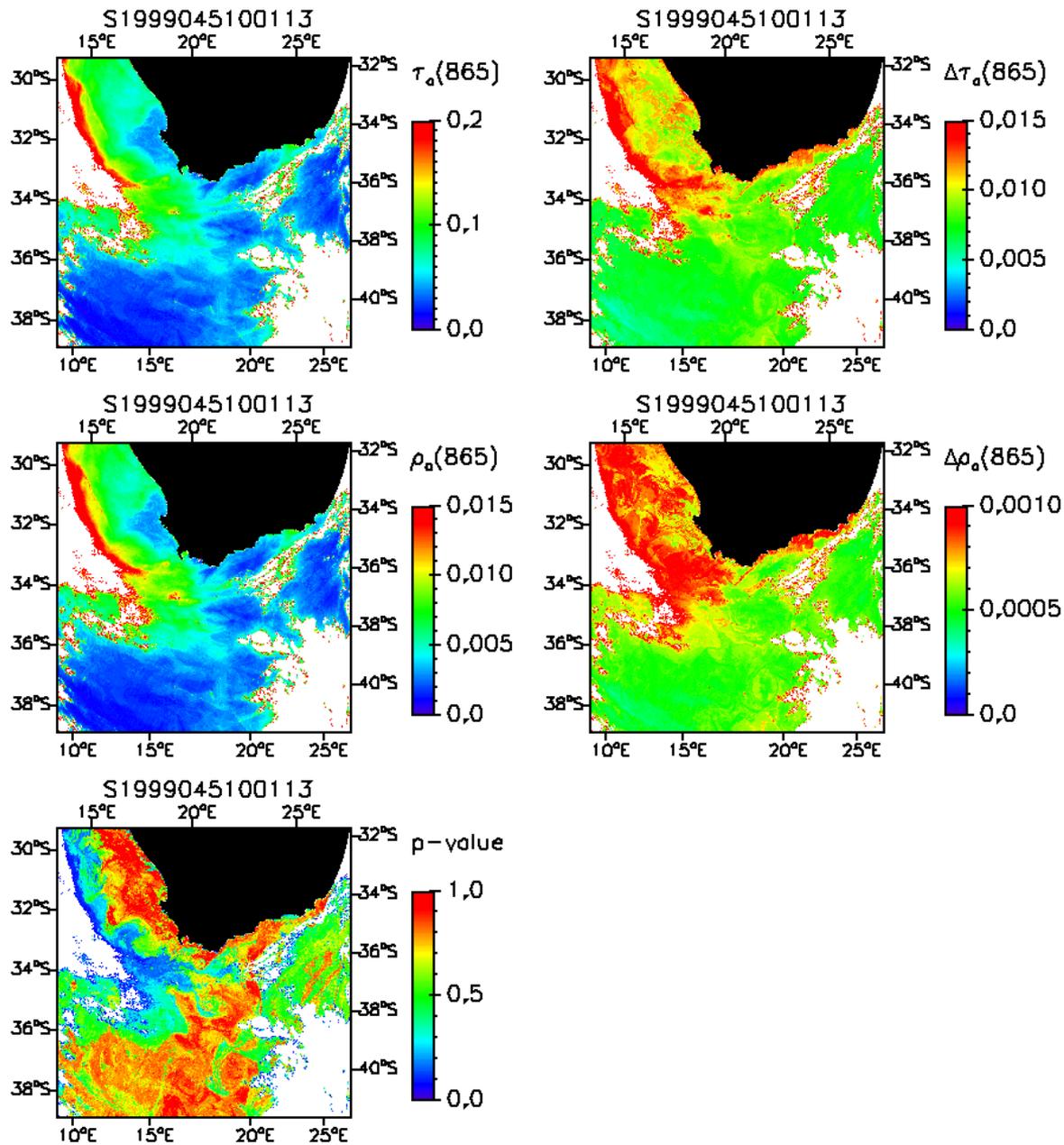
-This corresponds to inverting  $\rho$  assuming that the atmosphere is in the state  $x_a$ , and then averaging the results according to the distribution of  $x_a$  given  $\rho$ .

-So, compared with the classical approach, instead of picking an aerosol model and then inverting  $\rho$  assuming the atmosphere is in the state  $x_a$ , the Bayesian methodology amounts to placing a probability distribution on  $x_a$ , depending on  $\rho$ , inverting  $\rho$  for each  $x_a$ , and then averaging the results accordingly.

# Application to SeaWiFS Imagery, South Africa, 02/14/1999

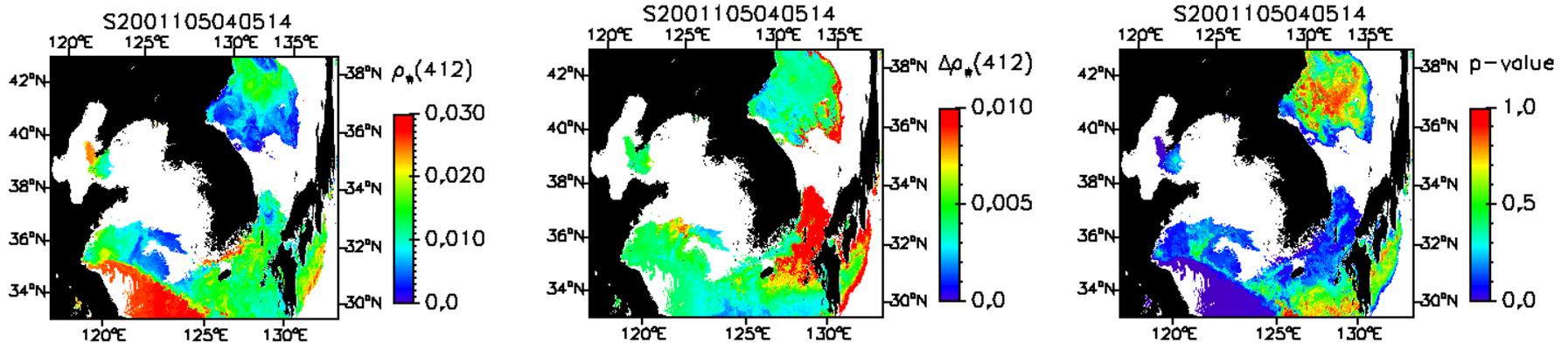


*Estimated  $\rho_w$  at 412, 490, and 555 nm and associated uncertainty, Bayesian methodology.*



*Estimated  $\tau_a$ ,  $\rho_a$  at 865 nm and associated uncertainty, and p-value, Bayesian methodology.*

# Application to SeaWiFS Imagery, East Asia Seas



*Estimated  $\rho_w$  at 412 nm, associated uncertainty, and p-value, Bayesian methodology. Uncertainty in East China Sea is relatively small (0.003-0.004), but p-value  $<0.01$ , indicating that model and observation are incompatible.*

## Conclusions

- The Bayesian approach is adapted to the ill-posed nature of the ocean color inverse problem.
- The solution, expressed as a probability distribution, allows the construction of reliable multi-dimensional confidence domains of the retrieved water reflectance.
- Expectation and covariance can be computed, which gives an estimate of the water reflectance and its uncertainty. The p-value identifies situations for which forward model and observation are incompatible.
- Covariance and p-value are complementary measures of uncertainty and quality. But they should be viewed in the context of a forward model. They do not replace uncertainties obtained by comparing retrievals with in situ measurements.