The Evolution of Radiative Transfer Theory 1729-2019

Curtis Mobley
Emeritus Vice President for Science
Sequoia Scientific, Inc.

IOCS, Busan, 2019

Everything presented today, and a lot more, is in
Notes on the Evolution of Radiative Transfer Theory
which can be downloaded as Mobley (2019) from the References/Publications page of the Ocean Optics Web Book at www.oceanopticsbook.info

For lack of time, there will be many omissions: Milne, Sobolev, Chandrasekhar, etc. Their contributions are discussed in the *Notes*.

There are many good stories to tell about each of the persons mentioned today, but time does not allow for these details. Again see the *Notes*.

Radiative Transfer Theory (RTT): Development or use of an equation that governs light propagation through an absorbing and/or scattering medium.

The Beginnings: Bouguer, Lambert, and Beer

Development of the Theory: Lommel to Preisendorfer

Completing the Bridge: Mishchenko

Pierre Bouguer (1698-1758)

French Professor of Hydrology
Father of Naval Architecture and Father of Photometry

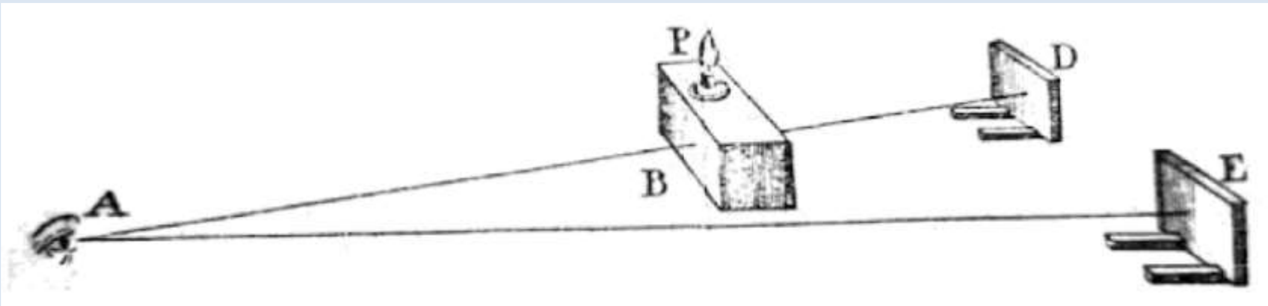


Plate 1, Fig. 5 from Bouguer, 1760

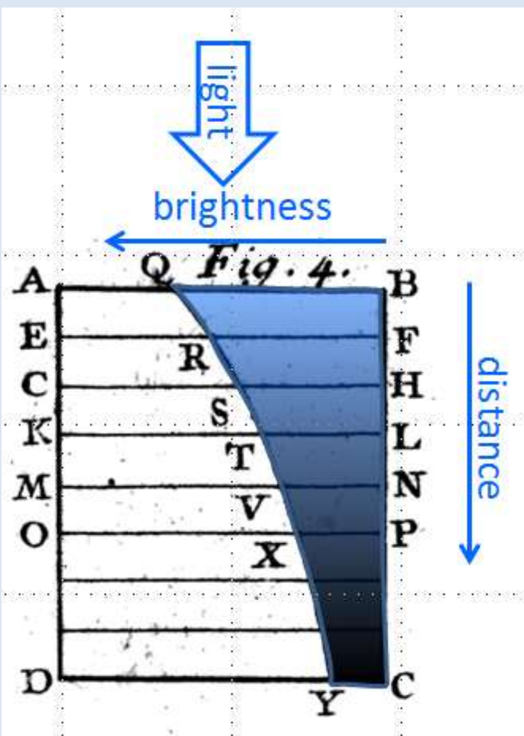
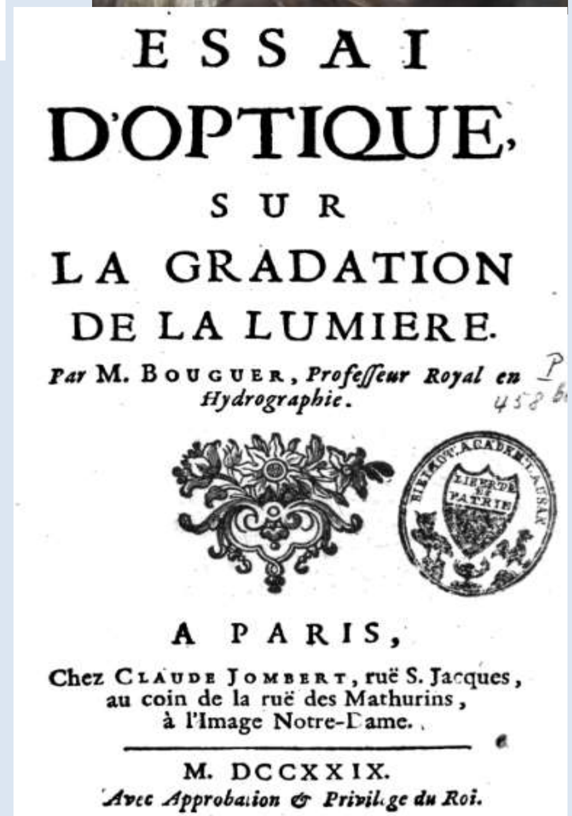


Figure 4 from Bouguer (1729), enhanced



ESSAI
D'OPTIQUE,
SUR
LA GRADATION
DE LA LUMIERE.
Par M. BOUGUER, *Professeur Royal en Hydrographie.*

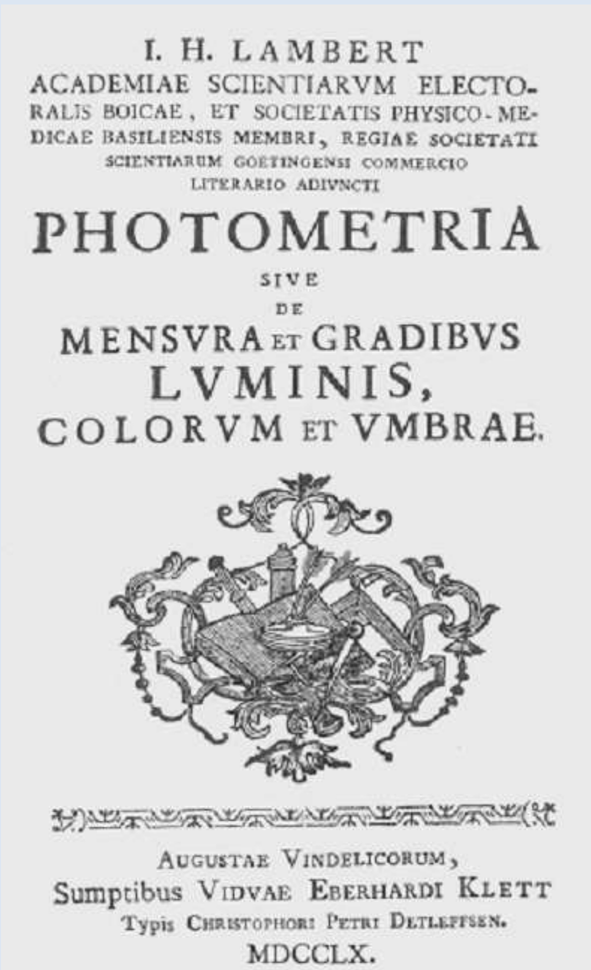
A PARIS,
Chez CLAUDE JOMBERT, ruë S. Jacques, au coin de la ruë des Mathurins, à l'Image Notre-Dame.

M. DCCXXIX.
Avec Approbation & Privilege du Roi.

1729

Johann Heinrich Lambert (1728-1777)

German mathematician
Cosine law for irradiance I(θ) = I(0) cos θ
Studied reflection by surfaces



I. H. LAMBERT
ACADEMIAE SCIENTIARVM ELECTORALIS BOICAE, ET SOCIETATIS PHYSICO-MEDICAE BASILIENSIS MEMBRI, REGIAE SOCIETATI SCIENTIARUM GOETINGENSI COMMERCIO LITERARIO ADIVNCTI

PHOTOMETRIA
SIVE
DE
MENSVRA ET GRADIBVS LVMINIS, COLORVM ET VMBRAE.

AUGUSTAE VINDELICORUM,
Sumptibus VIDVAE EBERHARDI KLETT
Typis CHRISTOPHORI PETRI DETLEFFSEN.
MDCCLX.

1760

$$E(x) = E(0)e^{-Kx}$$

$$dE(x)/\mathrm{dx} = -Kx$$

Lambert's Law: Exponential decrease with distance *x*, for a given material (*K*)

August Beer (1825-1863): "Determination of the absorption of red light in colored liquids" (1852)

Beer's Law: Exponential decrease with concentration (*K*) for a fixed distance *x*

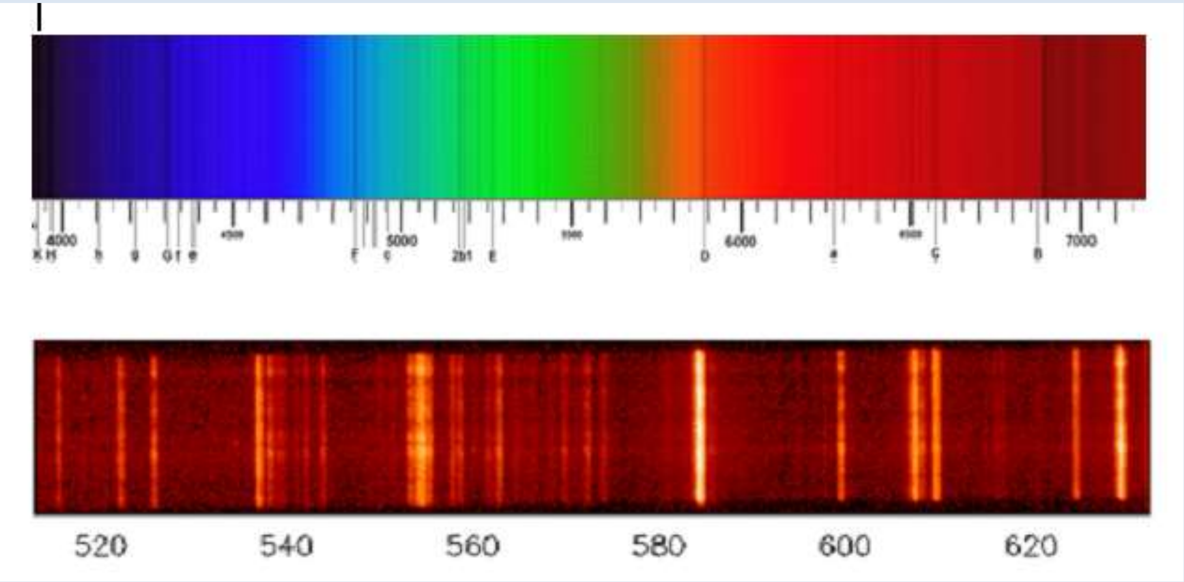
Three Research Topics in Astrophysics

Topic 1: Predicting the Albedo of Planets with Thick Clouds

“The photometry of diffuse backscatter”

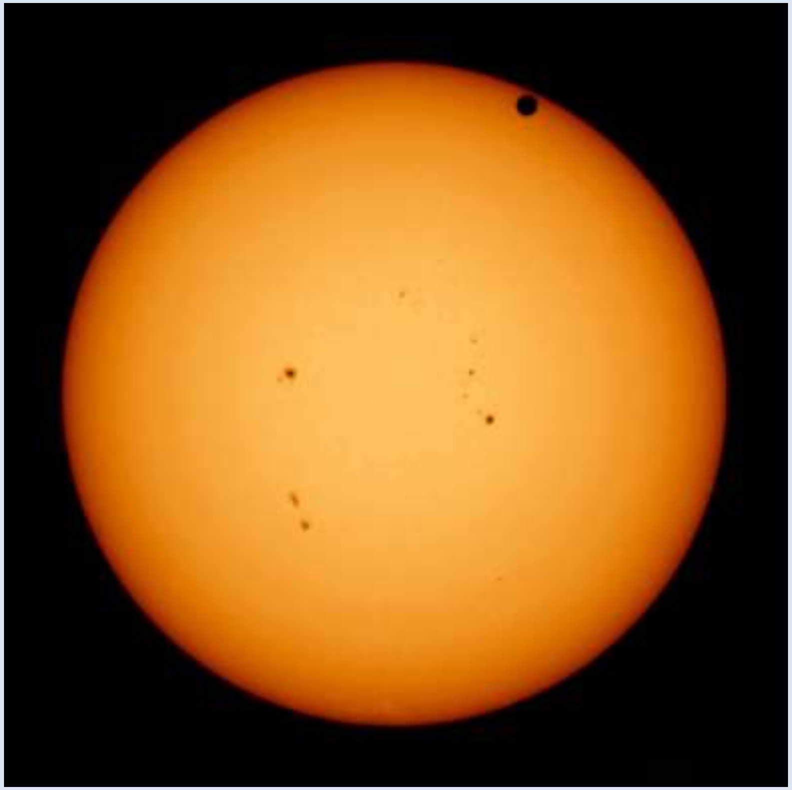
Sulphuric acid clouds on Venus

Topic 2: Understanding the Origin of Bright and Dark Lines in Solar Spectra



Solar spectra

Topic 3: Understanding Solar Limb Darkening

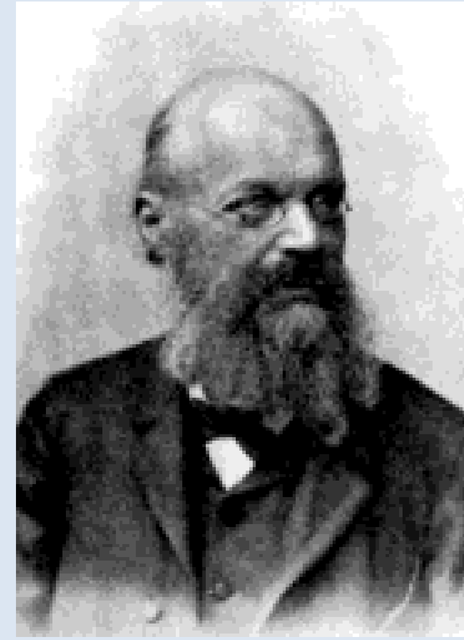


Both the brightness and the color change between the center and the edge (limb) of the Sun's disk.

Transit of Venus, 05 June 2012

Eugen Cornelius Joseph von Lommel
(1837-1899)

German physicist and mathematician



Die Photometrie der diffusen Zurückwerfung.

Von E. Lommel.

(Eingelaufen 7. Mai.)

In einer früheren Abhandlung „über Fluorescenz"[1]) habe ich in einem „über die Grundsätze der Photometrie" überschriebenen Abschnitt gezeigt, dass in der theoretischen Photometrie nicht, wie bis dahin üblich war, die Flächenelemente einer leuchtenden Oberfläche, sondern die Volumenelemente des leuchtenden Körpers als lichtstrahlend zu betrachten seien. Demgemäss wurden der theoretischen Behandlung photometrischer Probleme die folgenden drei Sätze zu Grunde gelegt:

1) Lommel, Wied. Ann. X. 449—472; 631—654. 1880.

"... in theoretical photometry it is not, as has been previously assumed, the surface element of a luminous surface that is to be considered, but rather the volume element of the luminous body that is to be regarded as the light emitting element."

1887

Lommel's 3 Axioms

1. The light amount (Lichtmenge) that is traveling from one volume element to another is proportional to the inverse square of the distance.

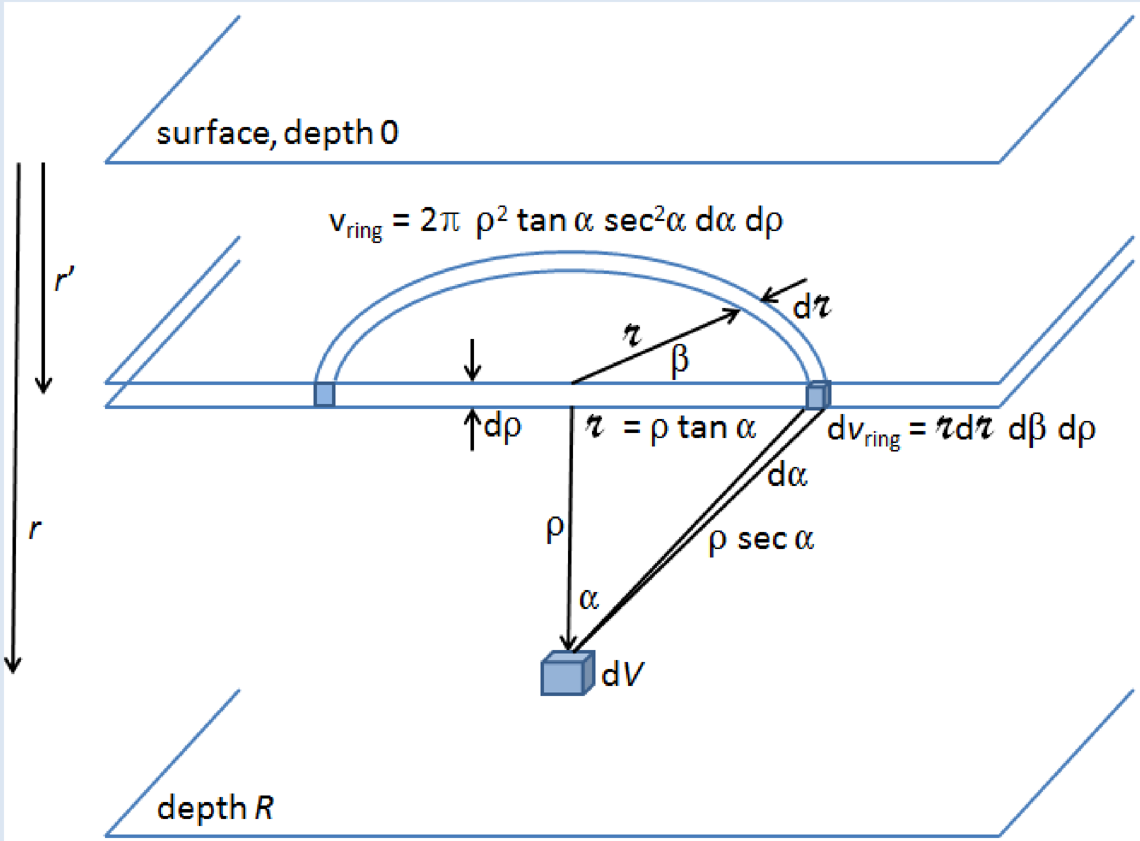
2. The light amount radiating from a volume element and falling onto a surface element is proportional to the cosine of the incident angle to the surface normal.

3. The light amount radiating from a volume element is decreased along the way by the absorption law.

He used these three axioms to derive an integral form of the RTE.

Assumed homogeneous medium, 1D geometry, isotropic scattering

Lommel's Geometry



$$dv_{ring} = r\, dr\, d\beta\, d\rho = (\rho \tan \alpha)\, (\rho \sec^2\alpha\, d\alpha)\, d\beta\, d\rho$$

$$dV \frac{F}{4\pi} \frac{2\pi\, \rho^2 \tan \alpha \sec^2 \alpha\, d\alpha\, d\rho}{(\rho \sec \alpha)^2}\, e^{-(k+\ell)\, \rho \sec \alpha}$$

F(*r′*) is the luminosity (Leuchtkraft)

Now integrate from depth r′ = 0 to R

Lommel's Integral Eq. for the "Light Amount" at Depth r

$$f(r) = -\frac{1}{2}\ell\left\{\int_0^r \left[a\,e^{-m r'/\cos i} + f(r')\right] li\left(e^{-m(r-r')}\right) dr' + \int_r^R \left[a\,e^{-m r'/\cos i} + f(r')\right] li\left(e^{-m(r'-r)}\right) dr'\right\}$$

$$li(x) \doteq \int_0^x \frac{dt}{\ln t}$$

$f(r)$ = "the total light amount per unit volume" at depth r
a = "the light amount of a parallel ray bundle" at $r = 0$
ℓ = the scattering coefficient
m = absorption coef + scattering coef
i = polar angle of the light incident onto the surface at $r = 0$

"The function $f(r)$ is of fundamental significance for theoretical photometry."

"It does not seem possible, with the mathematical tools now at our command, to obtain a solution in closed form."

Lommel’s Magnificent Achievement

- He recognized that the albedo of a medium depends on its internal absorption and scattering properties, not just on its surface reflectance properties.
- He made a clear distinction between the effects of absorption and scattering.
- He began with clearly stated axioms and derived an integral form of a radiative transfer equation (RTE) that is still readily recognized as such.
- He recognized that a small volume of material both receives scattered light and is the source of emitted light.
- He recognized the fundamental importance of his RTE.
- He formulated a solution algorithm for his integral equation that is now known as the successive-order-of-scattering technique.
- He used the single-scattering solution of his RTE to evaluate the albedo of a scattering and absorbing medium for a range of approximations.

Orest Danilovich Chwolson (Орест Данилович Хвольсон) (1852-1934)

Russian physicist

Grundzüge einer mathematischen Theorie der inneren Diffusion des Lichtes.
Von Dr. O. Chwolson. (Lu le 16 mai 1889)*).

*) Die vorliegende Abhandlung ist schon im Herbst 1885 Herrn Akademiker A. Gadolin und mir zur Vorstellung an die Akademie eingereicht worden, wurde aber dann vom Autor wieder zurückgezogen, um noch einen Versuch zur vollständigen Lösung der Hauptgleichung zu machen und dann fast unverändert im Herbst 1888 uns wieder zugestellt. Wegen des inzwischen erfolgten Erscheinens einer denselben Gegenstand behandelnden Abhandlung von E. Lommel (Wiedemann's Annalen Bd. 36) schien mir obige Erklärung nothwendig, welche ich leider in Abwesenheit meines Collegen Gadolin nur allein abgeben kann.

St. Petersburg, 4. Juli 1889. Akademiker *H. Wild.*

Bulletin de L'Académie Impériale des Sciences de St. Pétersbourg (1889)

"The present treatise was submitted to the Academy in the autumn of 1885, but was then withdrawn by the author in order to make another attempt at a complete solution of the main equation, and was then resubmitted almost without change in the autumn of 1888. Because of the appearance in the meantime of a treatise dealing with the same subject by Lommel, the above explanation seemed necessary to me." – H. Wild

Chwolson's analysis was very similar to Lommel's

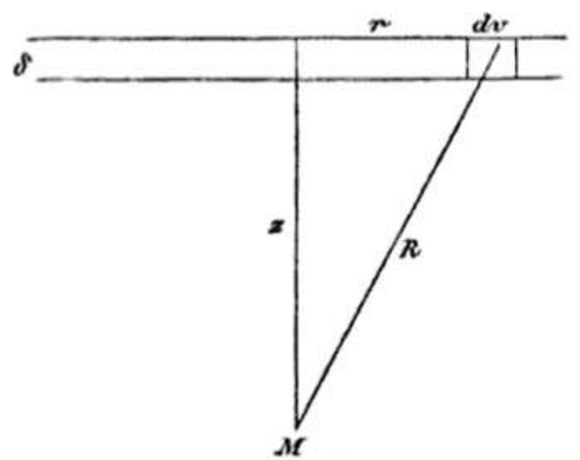


Fig. 2.

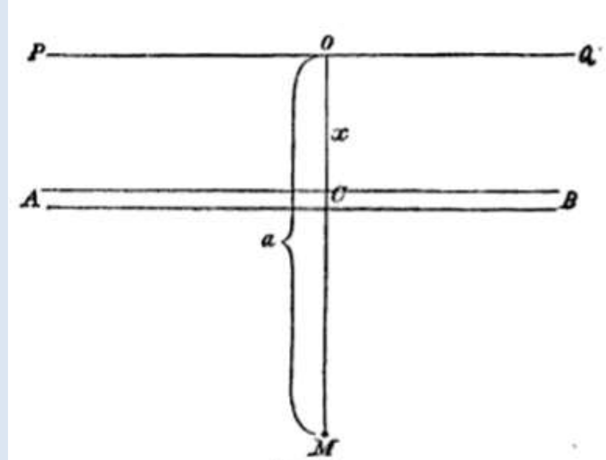
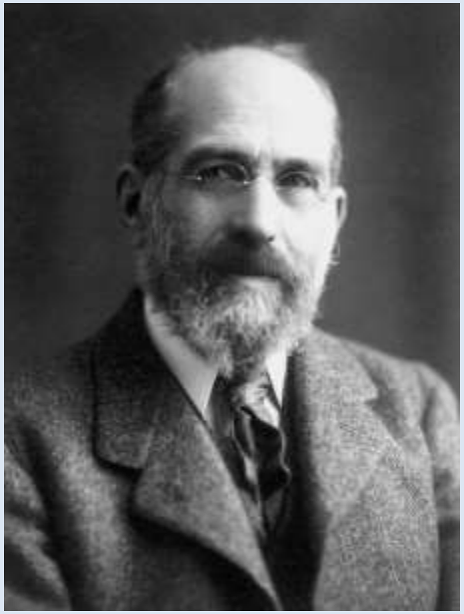


Fig. 3.

$$f(a) = e^{-pa} - \frac{\alpha K}{2}\int_0^a f(x)\,\omega(pa-px)\,dx - \frac{\alpha K}{2}\int_a^h f(x)\,\omega(px-pa)\,dx$$

Essentially the same as Lommel's equation

Franz Arthur Friedrich Schuster (1851-1934)



German-English physicist

"Radiation through a foggy atmosphere" (1905)
Developed the two-flow irradiance equations by heuristic arguments and used them to explain bright vs dark absorption lines in the Sun's spectrum as function of the relative amounts of absorption vs scattering, and the depth profile of the blackbody emission (i.e, depth profile of temperature).

$$\frac{dA}{dx} = \kappa(E - A) + \tfrac{1}{2}s(B - A)$$

$$\frac{dB}{dx} = \kappa(B - E) + \tfrac{1}{2}s(B - A)$$

The two-flow equations for isotropic scattering and an isotropic internal source

A is E_d and B is E_u
κ is the absorption coef
s/2 is the backscatter coef
κE is blackbody emission in the solar atmosphere

Schuster (1905) is often cited as the beginning of RTT (by those unfamiliar with Lommel and Chwolson).

Max Karl Ernst Ludwig Planck (1858-1947)

German physicist; Nobel Prize 1918

fortpflanzt. Die Öffnung dieses Kegels ist:

$$d\,\Omega = \sin\vartheta \cdot d\,\vartheta \cdot d\,\varphi\,. \qquad (5)$$

Auf diese Weise erhalten wir für die Energie, welche in der Zeit $d\,t$ durch das Flächenelement $d\,\sigma$ in der Richtung des Kegels $d\,\Omega$ hindurchgestrahlt wird, den Ausdruck:

$$dt\,d\sigma\,\cos\vartheta\,d\,\Omega\,K = K\sin\vartheta\,\cos\vartheta\,d\vartheta\,d\varphi\,d\sigma\,dt\,. \qquad (6)$$

Die endliche Größe K nennen wir die „spezifische Intensität" oder auch die „Helligkeit", $d\,\Omega$ den „Öffnungswinkel" des von einem Punkte des Elementes $d\,\sigma$ in der Richtung (ϑ, φ) ausgehenden Strahlenbündels. K ist eine positive Funktion des Ortes, der Zeit und der beiden Winkel ϑ und φ. Die spezifischen Strahlungsintensitäten nach verschiedenen Richtungen sind im allgemeinen gänzlich unabhängig voneinander. Setzt man z. B. in der Funktion K für ϑ den Wert $\pi - \vartheta$, und für φ den Wert $\pi + \varphi$, so erhält man die spezifische Strahlungsintensität in der gerade entgegengesetzten Richtung, eine im allgemeinen von der vorigen ganz verschiedene Größe.

Die Gesamtstrahlung durch das Flächenelement $d\,\sigma$ nach einer Seite, etwa derjenigen, für welche der Winkel ϑ ein spitzer ist, ergibt sich durch Integration über φ von 0 bis $2\,\pi$, und über ϑ von 0 bis $\frac{\pi}{2}$:

$$\int_0^{2\pi} d\,\varphi \int_0^{\pi/2} d\,\vartheta\; K\sin\vartheta\,\cos\vartheta\,d\sigma\,dt\,.$$

Ist die Strahlung nach allen Richtungen gleichmäßig, also K konstant, so folgt hieraus für die Gesamtstrahlung durch $d\,\sigma$ nach einer Seite:

$$\pi\,K\,d\sigma\,dt\,. \qquad (7)$$

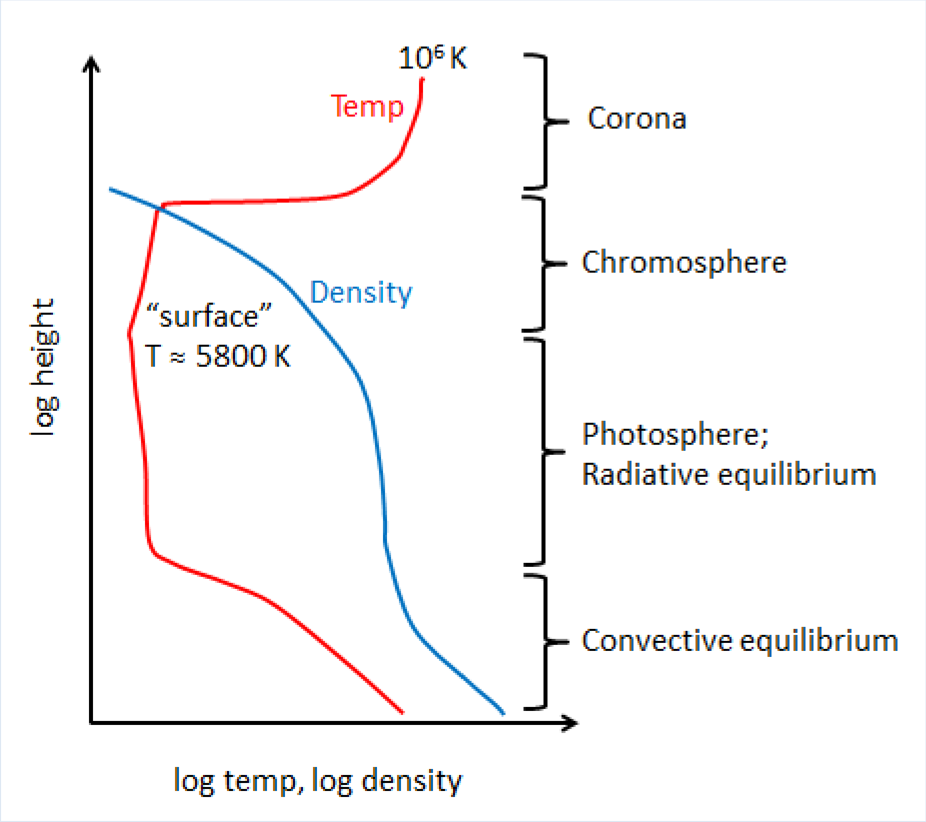
First formulation of radiance ("specific intensity") as used today in *Lectures on the Theory of Heat Radiation* (1906)

Page 15 of *Vorlesungen über die Theorie der Wärmestrahlung* (1906)

Karl Schwarzschild (1873-1916)

German physicist and astronomer

"On the equilibrium of the solar atmosphere" (1906) introduced the concept of radiative equilibrium to explain the Sun's limb darkening.



Radiative equilibrium: energy transport is by thermal radiation (Sun's photosphere)

Convective equilibrium: energy transport is by convective mixing (Earth's atmosphere & deeper in the Sun)

Formulated an RTE for radiance for absorption and emission. Assumed a temperature profile to get blackbody emission profile $E(\mu)$. Then could solve for angular distribution of radiance $F(i)$ leaving the Sun's photosphere. Compared his $F(i)$ for with observation to show that the photosphere is in radiative equilibrium. He thus solved an inverse RT problem to deduce the Sun's temperature profile from the angular pattern of emitted radiance.

Strahlung. Man erhält daher für F in völliger Analogie zu (8) die Differentialgleichung:

$$\frac{dF}{dh} = -\frac{\alpha}{\cos i}(E-F) \tag{18}$$

oder:

$$\frac{dF}{d\mu} = -\frac{1}{\cos i}(E-F),$$

wenn man die Abkürzung:

$$\mu = \int_{-\infty}^{h} \alpha\, dh \tag{19}$$

einführt. Die Integration ergiebt für die aus der Atmosphäre austretende Strahlung:

$$F(i) = \int_0^\infty E e^{-\mu \sec i} d\mu \sec i. \tag{20}$$

Es läßt sich also $F(i)$ berechnen, sobald man die Temperaturverteilung längs der Vertikalen und damit E als Funktion von μ kennt.

		Measured	Radiative equilibrium	Convective equilibrium
	$\frac{r}{R}$	Messung.	Strahlungs-gleichgew.	Adiabat. Gleichgew.
Sun's center	0.0	1.00	1.00	1.00
	0.2	0.99	0.99	0.98
	0.4	0.97	0.95	0.92
	0.6	0.92	0.87	0.80
	0.7	0.87	0.81	0.71
	0.8	0.81	0.73	0.60
	0.9	0.70	0.63	0.44
	0.96	0.59	0.52	0.28
	0.98	0.49	0.47	0.20
Sun's limb	1.00	(0.40)	0.33	0.00

"Ueber das Gleichgewicht der Sonnenatmosphäre" (1906)

Louis Vessot King (1886-1956)

Canadian physicist, mathematician, and inventor

"On the Scattering and Absorption of Light in Gaseous Media, with Applications to the Intensity of Sky Radiation" (1913) "Each element of volume will scatter a certain proportion of the radiation incident upon it, so that each element besides being illuminated by the incident radiation is also subject to the aggregate radiation from all the other elements within the surface Σ, i.e., to the effect of *self-illumination*." (italics his).

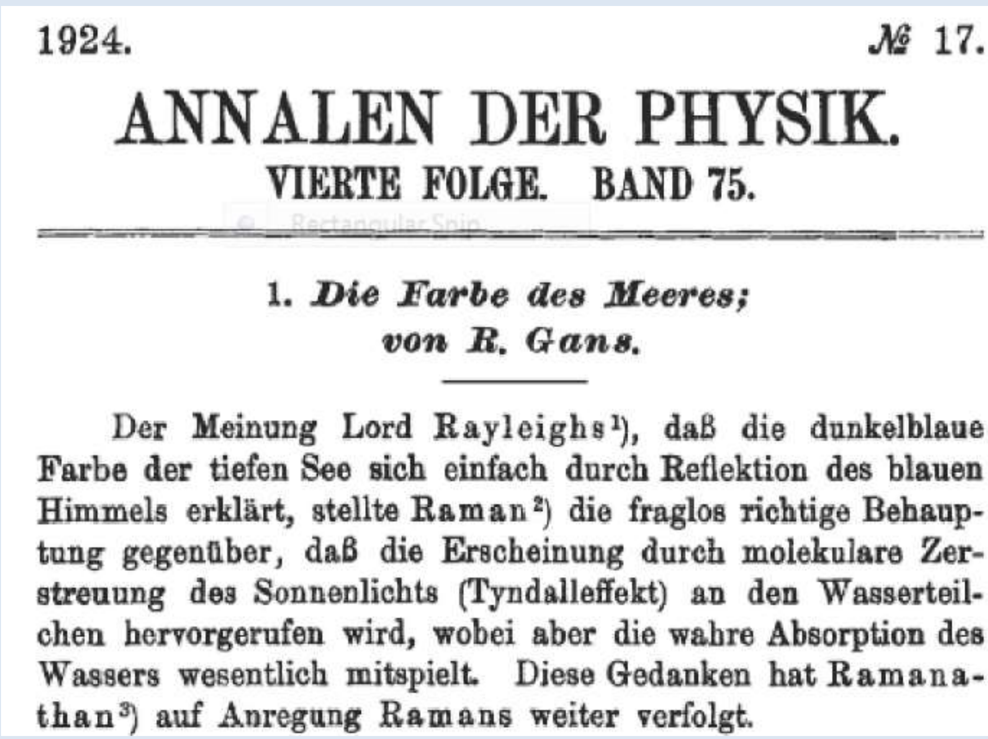
obtain the following *integral equation* for the scattered radiation at and from any point,

$$\mathrm{I}(x, y, z, 0, \theta) = \mu(\theta)\,\mathrm{E}(x, y, z) + \int_{\Sigma} \mu(\widehat{rr'})\,\mathrm{I}(x', y', z', 0, \theta')\, r'^{-2} e^{-\int_0^{r'} \mathrm{K}\, dr'}\, dv', \quad . \qquad (14)$$

First formulation of an RTE for arbitrary phase function.
Footnote that Lommel's problem "...is included as a particular case of the investigation of the present paper." (The only Lommel reference until 1980)

Richard Martin Gans (1880-1954)
German who lived in Argentina

"The Color of the Sea" (1924) is the first paper to apply RTT to the ocean.



1924. № 17.

ANNALEN DER PHYSIK.
VIERTE FOLGE. BAND 75.

1. *Die Farbe des Meeres;*
von R. Gans.

Der Meinung Lord Rayleighs[1]), daß die dunkelblaue Farbe der tiefen See sich einfach durch Reflektion des blauen Himmels erklärt, stellte Raman[2]) die fraglos richtige Behauptung gegenüber, daß die Erscheinung durch molekulare Zerstreuung des Sonnenlichts (Tyndalleffekt) an den Wasserteilchen hervorgerufen wird, wobei aber die wahre Absorption des Wassers wesentlich mitspielt. Diese Gedanken hat Ramanathan[3]) auf Anregung Ramans weiter verfolgt.

Opens with Rayleigh's infamous quote, "The much-admired dark blue of the deep sea has nothing to do with the colour of water, but is simply the blue of the sky seen by reflection."

Gans worked entirely with electric fields (Maxwell's equations) and included polarization parallel and perpendicular to the meridian plane.

ist, so ergibt sich für die spezifische Intensität K in Luft:

$$(22)\quad \begin{cases} K_s = \dfrac{3}{16\pi}\,\dfrac{h}{h+h'}\,\dfrac{J_0\, d_p(\alpha)\, d_s(\Theta)}{n^2}\,\dfrac{\cos\beta\cos^2\vartheta\sin^2\varphi}{\cos\beta+\cos\vartheta}, \\[2ex] K_p = \dfrac{3}{16\pi}\,\dfrac{h}{h+h'}\,\dfrac{J_0\, d_p(\alpha)\, d_p(\Theta)}{n^2}\,\dfrac{\cos\beta\cos^2\varphi}{\cos\beta+\cos\vartheta}. \end{cases} \qquad \text{(Fall } p\text{)}$$

K_s, K_p = radiance perpendicular, parallel to meridian plane for incident radiance parallel

h = scattering coef (assumed independent of wavelength)

h' = absorption coef, a function of wavelength

J_o = incident radiance spectrum

n = index of refraction of water (weak wavelength dependence)

The wavelength dependence is in the absorption coef h'.

"We can therefore say: *That the ocean sends out diffuse light at all is due to molecular light scattering; that this light is blue is explained by the true absorption in the red, yellow, and blue.*" (his italics)

Andrei Aleksandrovich Gershun (1903-1952)
(Андрей Александрович Гершун)

Russian physicist, Father of Lighting Technology and of Soviet optical oceanography

"Fundamental Ideas of the Theory of Light Fields (Vector Methods of Light Calculations)" (1936)

А. А. ГЕРШУН

ОСНОВНЫЕ ПРЕДСТАВЛЕНИЯ ТЕОРИИ СВЕТОВОГО ПОЛЯ

(Векторные методы светотехнического расчета)

$$\operatorname{div}\vec{\mathcal{E}} = -k\mathcal{E}_0,$$

где k — коэффициент поглощения [7].

"The space density of light, produced (or absorbed) per unit time, is equal to the divergence of the light vector." (his italics)

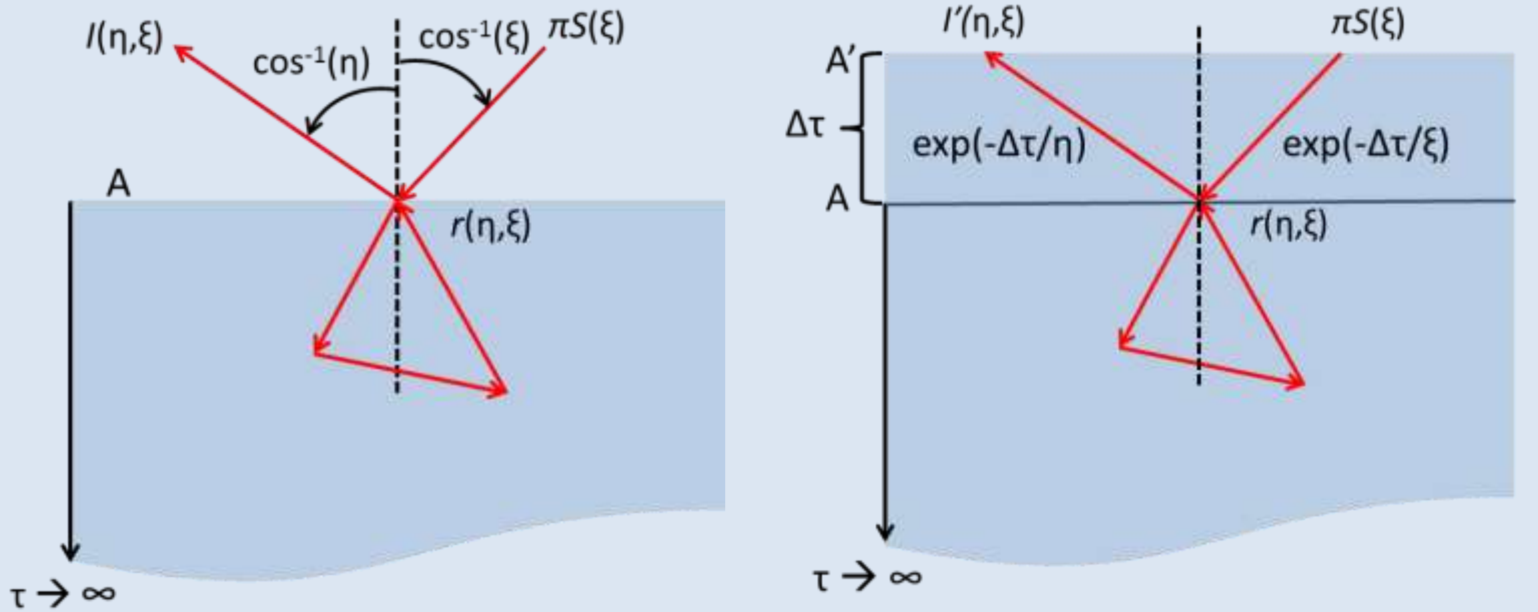
Gershuns's 1936 book *The Light Field* (светового поля) is of historical significance for the development of RTT because *it was the first serious attempt to place radiometry on a firm physical foundation. That is to say, its goal was to make a previously phenomenological theory a part of physics.*

Victor Amazaspovich Ambartsumian (1908-1996)
(Виктор Амазаспович Амбарцумян)

Armenian/Soviet astrophysicist; today a national hero in Armenia

1943 paper: "On the issue of diffuse reflection of light by a turbid medium"

$$r(\eta,\xi) = \frac{I(\eta,\xi)}{S(\xi)}$$



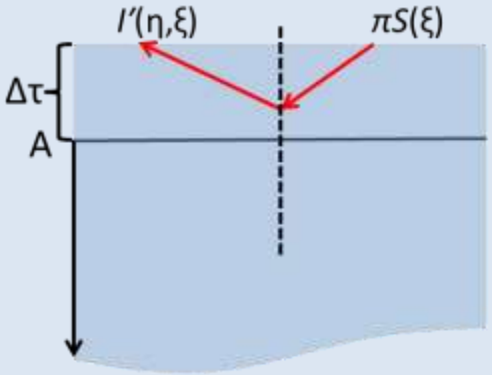
(A) The original problem.

(B) The medium with a thin layer added.

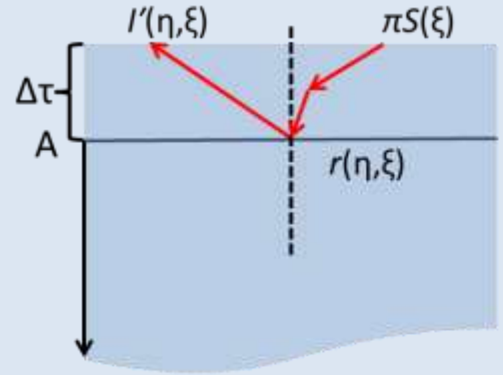
$$\begin{aligned} I'(\eta,\xi) &= \exp(-\Delta\tau/\eta)\, r(\eta,\xi)\, \exp(-\Delta\tau/\xi)\, S \\ &= r(\eta,\xi)\left(1 - \frac{\Delta\tau}{\eta} + \cdots\right)\left(1 - \frac{\Delta\tau}{\xi} + \cdots\right) S \\ &\approx r(\eta,\xi)\left(1 - \frac{\Delta\tau}{\eta} - \frac{\Delta\tau}{\xi}\right) S, \end{aligned}$$

Scattering interactions of order Δτ in the added layer

$$\frac{\lambda}{4}\,\frac{\Delta\tau}{\eta}\,S$$



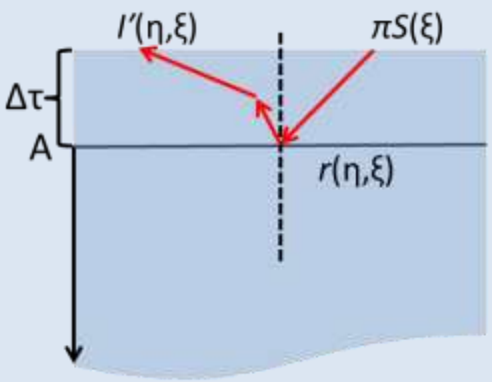
Case 1: Scattering within the added layer without reflection at A



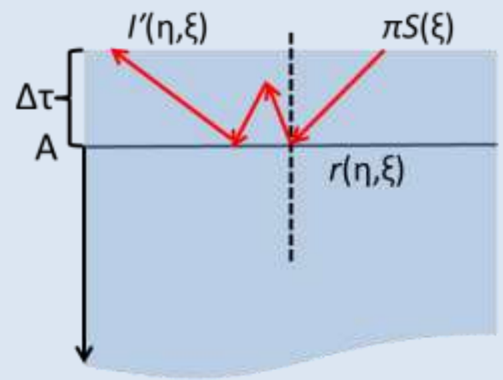
Case 2: Scattering within the added layer and then reflection at A

$$\frac{\lambda}{2}\,\Delta\tau\,S\int_0^1 r(\eta,\zeta)\,\frac{d\zeta}{\zeta}$$

$$\frac{\lambda}{2}\,\frac{\Delta\tau}{\eta}\,S\int_0^1 r(\zeta,\xi)\,d\zeta$$



Case 3: Reflection at A and then scattering within the added layer



Case 4: Reflection at A, scattering, and then reflection at A again

$$\lambda\,\Delta\tau\,S\int_0^1 r(\zeta,\xi)\,d\zeta\int_0^1 r(\eta,\zeta')\,\frac{d\zeta'}{\zeta'}$$

λ is the scattering coef. (beam c = 1); isotropic scatter is assumed.
Other interactions are of higher order in Δτ and are ignored

Now add up these 5 contributions, and observe that ***the reflected radiance with the added layer, I'(η,ξ), is the same as the original I(η,ξ)*** **[Ambartsumian's Principle of Invariance]*:***

$$r(\eta,\xi) = r(\eta,\xi)\left(1 - \frac{\Delta\tau}{\eta} - \frac{\Delta\tau}{\xi}\right) + \frac{\lambda}{4}\frac{\Delta\tau}{\eta} + \frac{\lambda}{2}\Delta\tau\int_0^1 r(\eta,\zeta)\,\frac{d\zeta}{\zeta}$$
$$+ \frac{\lambda}{2}\frac{\Delta\tau}{\eta}\int_0^1 r(\zeta,\xi)\,d\zeta + \lambda\,\Delta\tau\int_0^1 r(\zeta,\xi)\,d\zeta\int_0^1 r(\eta,\zeta')\,\frac{d\zeta'}{\zeta'}$$

$$\left(\frac{1}{\eta} + \frac{1}{\xi}\right) r(\eta,\xi) = \frac{\lambda}{4}\left[\frac{1}{\eta} + 2\int_0^1 r(\eta,\zeta)\,\frac{d\zeta}{\zeta} + \frac{2}{\eta}\int_0^1 r(\zeta,\xi)\,d\zeta + 4\int_0^1 r(\zeta,\xi)\,d\zeta\int_0^1 r(\eta,\zeta')\,\frac{d\zeta'}{\zeta'}\right]$$

This is an integral functional equation for the unknown reflectance *r*(η,ξ), given the IOPs via the scattering coefficient λ and the assumed isotropic phase function, and the desired incident and final angles. ***This equation allows the computation of the radiance reflectance of the optically deep medium without the need to solve the radiative transfer equation within the medium!***

Введем функцию $R(\eta, \xi)$, определенную через

$$r(\eta, \xi) = \frac{\lambda}{4\eta} R(\eta, \xi). \tag{1}$$

Тогда для $R(\eta, \xi)$ имеем функциональное уравнение

$$\left(\frac{1}{\eta} + \frac{1}{\xi}\right) R(\eta, \xi) = 1 + \frac{\lambda}{2}\int_0^1 R(\eta, \zeta)\frac{d\zeta}{\zeta} + \frac{\lambda}{2}\int_0^1 R(\zeta, \xi)\frac{d\zeta}{\zeta} + \frac{\lambda^2}{4}\int_0^1 R(\eta, \zeta')\frac{d\zeta'}{\zeta'}\int_0^1 R(\zeta, \xi)\frac{d\zeta}{\zeta}. \tag{2}$$

Очевидно, что если этому уравнению удовлетворяет функция $R(\eta, \xi)$, то ему же должна удовлетворять функция $R(\xi, \eta)$. Так как наша физическая задача должна иметь только одно решение, то возникает мысль искать решение уравнения (2) в виде симметричной функции

$$R(\eta, \xi) = R(\xi, \eta). \tag{3}$$

Но при этом условии правая часть (2) оказывается произведением двух одинаковых функций

$$\left(\frac{1}{\eta} + \frac{1}{\xi}\right) R(\eta, \xi) = \left\{1 + \frac{\lambda}{2}\int_0^1 R(\eta, \zeta)\frac{d\zeta}{\zeta}\right\}\left\{1 + \frac{\lambda}{2}\int_0^1 R(\xi, \zeta)\frac{d\zeta}{\zeta}\right\}. \tag{4}$$

Обозначим

$$\varphi(\eta) = 1 + \frac{\lambda}{2}\int_0^1 R(\eta, \zeta)\frac{d\zeta}{\zeta}. \tag{5}$$

Тогда (4) и (5) сразу дают структуру функций $R(\eta, \xi)$ и $r(\eta, \xi)$:

$$R(\eta, \xi) = \frac{\varphi(\eta)\varphi(\xi)}{\frac{1}{\eta} + \frac{1}{\xi}}; \quad r(\eta, \xi) = \frac{\lambda}{4}\xi\frac{\varphi(\eta)\varphi(\xi)}{\eta + \xi}. \tag{6}$$

Подстановка (6) в (5) дает уравнение для функции $\varphi(\eta)$

$$\varphi(\eta) = 1 + \frac{\lambda}{2}\eta\varphi(\eta)\int_0^1 \frac{\varphi(\xi)\, d\xi}{\eta + \xi}. \tag{7}$$

Итак, мы приходим к выводу: функция $r(\eta, \xi)$, характеризующая отражательную способность, имеет структуру, выражаемую формулой (6). Функция $\varphi(\eta)$ определяется при этом функциональным уравнением (7).

One of the most important pages in the history of RTT

$$\varphi(\eta) = 1 + \frac{\lambda}{2}\eta\,\varphi(\eta)\int_0^1 \frac{\varphi(\xi)\, d\xi}{\eta + \xi}$$

$$R(\eta,\xi) = \frac{\varphi(\eta)\,\varphi(\xi)}{\frac{1}{\eta} + \frac{1}{\xi}}$$

$$R(\eta,\xi) = \frac{4\eta}{\lambda}\, r(\eta,\xi)$$

This page is the beginning of HydroLight

Rudolph William Preisendorfer (1927-1986)

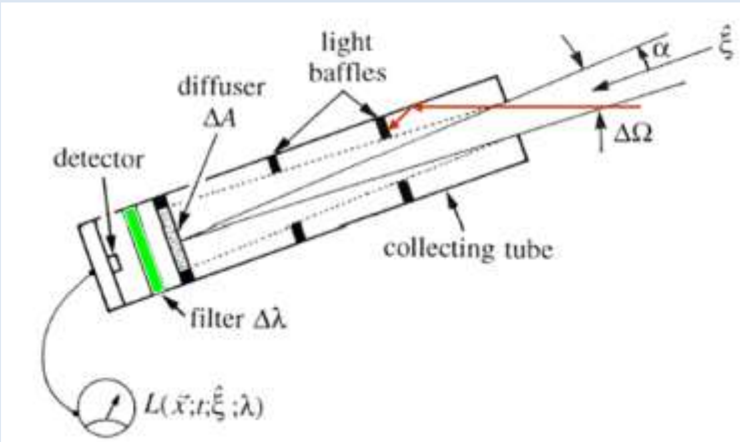
American applied mathematician

(1) Set himself the goal of constructing "an analytical bridge between the mainland of physics and the island of radiative transfer theory"
(2) Generalized Ambartsumian's idea to include radiance reflection and transmission by inhomogeneous media, arbitrary phase functions, finitely thick layers—just what is needed for oceanography. Now called "Invariant Imbedding Theory."

No Preisendorfer ⟹ No HydroLight

Fundamental Problems with Phenomenological RTT

The usual derivation of the RTE is based on **energy** arguments.



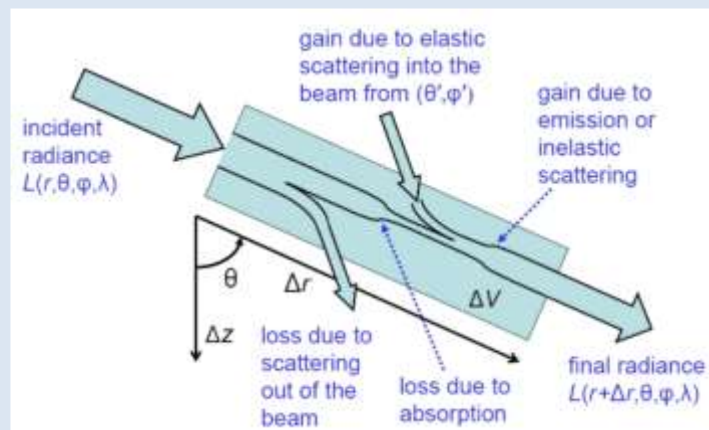
$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\Delta Q}{\Delta t\, \Delta A\, \Delta \Omega\, \Delta \lambda}$$

$$(\mathrm{J\ s^{-1}\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

$$(\mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}})$$

Radiance is usually described as giving the "angular distribution of radiant (electromagnetic, light) energy flow."

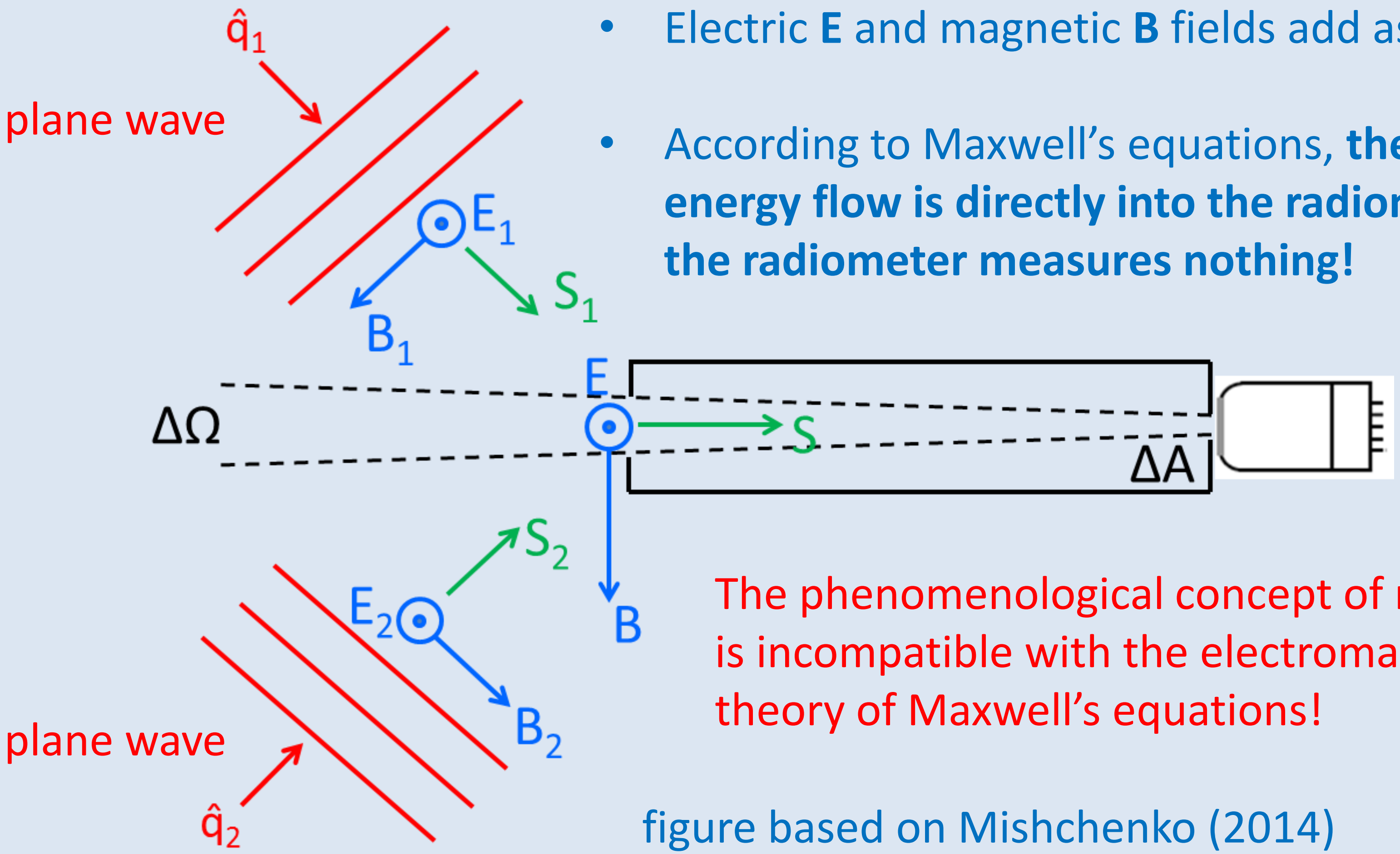
Then you make arguments about conservation of energy in the form of radiance, and get an RTE.



The fundamental problem with this formulation is that matter does not interact with *energy*, it interacts with *electric and magnetic fields*.

What Does a Radiometer Measure?

- Energy propagates in direction $\mathbf{S} = (1/\mu_o)\,\mathbf{E} \times \mathbf{B}$ (the Poynting vector) [**S** has units of irradiance]
- Electric **E** and magnetic **B** fields add as vectors
- According to Maxwell's equations, **the net energy flow is directly into the radiometer, but the radiometer measures nothing!**



The phenomenological concept of radiance is incompatible with the electromagnetic theory of Maxwell's equations!

figure based on Mishchenko (2014)

What is Needed to Make RTT a Part of Physics

Quantum Electrodynamics (QED)

↓ Extremely difficult, but done by R. P. Feynman

Maxwell's Equations

↓ The missing link, recently completed by M. I. Mishchenko

The most general vector RTE

↓ Requires a Ph.D. in Physics

VRTE for particles with mirror symmetry

↓ Easy, see the RTT chapter of the Ocean Optics Web Book

Scalar RTE from the first component of the VRTE
(what HydroLight solves)

↓ Trivial, see *Light and Water*

Everything else (irradiances, reflectances, etc.)

Michael I. Mishchenko

Senior Scientist at the NASA Goddard
Institute of Space Studies

In practice we need to solve two well posed physical problems:

(1) Need to compute the radiant energy budget of a volume of material (say a volume of ocean water). This will allow us to compute the heating of the water, and to compute how much light energy is available for photosynthesis.

(2) Need to understand exactly what a “well collimated radiometer” measures. This will allow us to properly interpret our measurements and to understand the relation between what is in the water and the optical quantities being measured, which is the basis of remote sensing.

Mishchenko shows that both of these problems can be solved **without resorting to heuristic arguments or even introducing the concept of radiance**.

Two Paths to the SRTE

but two levels of understanding and two different interpretations

Rigorous Derivation

Maxwell's Equations (light described by electric and magnetic fields)

very difficult physics and math, but **<u>no energy arguments, and no mention of radiance</u>**

The general vector RTE

various simplifications and approximations

Phenomenological Derivation

Define radiance

heuristic arguments about conservation of energy as expressed by the radiance

The scalar RTE as solved by HydroLight

Your Homework Assignment

A good place to start understanding the deficiencies of phenomenological RTT and Mishchenko's contributions to RTT is his two recent reviews:

"125 Years of Radiative Transfer: Enduring Triumphs and Persisting Misconceptions" (Mishchenko, 2013)

"Directional radiometry and radiative transfer: The convoluted path from centuries-old phenomenology to physical optics" (Mishchenko, 2014).

Even if you can't follow the math, you need to read these two papers to get an ideal of the fundamental problems with phenomenological RTT, and of the resolution of those problems.

The Highlights

- Lommel (1887) and Chwolson (1889) wrote seminal papers, but were soon forgotten. It was decades before physics and math of equal sophistication again appeared in RTT in King (1913)
- Schuster (1905) and Schwarzshild (1906) (and others) used simple equations to obtain approximate solutions to a variety of astrophysical problems.
- Gans (1924) was the first to apply RTT to the ocean, the first to consider polarization, and the first to use electromagnetic theory in RTT.
- Ambartsumian (1943) found an entirely new approach to the reflectance problem of Lommel and Chwolson. Fortunately Ambartsumian did not suffer the same fate of obscurity as did Lommel and Chwolson.
- Preisendorfer (1960s & 1970s) generalized Ambartsumian’s idea.
- Mishchenko (2000s) completed the bridge between fundamental physics and RTT.

My nominations:

- Lommel and Chwolson should be called “The Fathers of Radiative Transfer Theory”
- Mishchenko gets the title of “Deepest Thinker in Radiative Transfer Theory”

Sea Kayaking in Antarctica, Feb-Mar 2018

For the full story, see ann-and-curt.smugmug.com